Building a Calculator: Foundation for Database Development

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Abstract

This article explores the fundamental connection between calculator development and database engine construction. By implementing a recursive descent parser for mathematical expressions, we establish core patterns that directly translate to SQL query processing, expression evaluation, and database optimization. The calculator serves as a concrete foundation for understanding tokenization, parsing, and evaluation techniques essential for database development.

1 Introduction: From Arithmetic to SQL

The journey from building a simple calculator to developing a sophisticated database engine may seem like a significant leap, but the fundamental concepts are remarkably similar. When a database executes `SELECT price * quantity FROM orders WHERE total > 100`, it performs the same core operations as our calculator: tokenizing input, parsing expressions according to precedence rules, and evaluating results.

This article demonstrates how implementing a recursive descent parser for mathematical expressions provides the foundation for SQL query parsers used in production databases. The expression evaluation engine we develop becomes the cornerstone for computing derived columns, WHERE clause conditions, and aggregate functions. By mastering these concepts in the familiar context of arithmetic, we prepare ourselves for the more complex challenges of database system development.

2 Problem Definition and Requirements

A robust calculator must handle mathematical expressions with proper operator precedence, parentheses, and comprehensive error handling. Consider the expression $3 + 4 \times 2$. A naive left-to-right evaluation yields 14, but mathematical convention demands 11. This precedence handling mirrors how databases must parse `SELECT * FROM users WHERE age > 18 AND status = 'active'`, ensuring logical operators are evaluated correctly.

Our calculator implementation will support:

- Basic arithmetic operators ($+$, $-$, $\times$, $\div$)
- Parentheses for expression grouping
- Floating-point number handling
- Robust error handling for division by zero and syntax errors
- Extensible architecture for future function additions

3 Recursive Descent Parser Implementation

The recursive descent parser employs a top-down parsing technique that mirrors the grammatical structure of mathematical expressions. Each grammar rule corresponds to a function, and the parser recursively calls these functions to construct an expression tree.

3.1 Grammar Specification

Our calculator follows this formal grammar hierarchy:

$$\text{expression} \rightarrow \text{term } (('+' \mid '-') \text{ term})^* \tag{1}$$

$$\text{term} \rightarrow \text{factor } (('*' \mid '/') \text{ factor})^* \tag{2}$$

$$\text{factor} \rightarrow \text{number} \mid '(' \text{ expression } ')' \tag{3}$$

$$\text{number} \rightarrow \text{digit}^+('.'\text{digit}^+)? \tag{4}$$

This grammar naturally encodes operator precedence: expressions handle addition and subtraction (lowest precedence), terms handle multiplication and division (higher precedence), and factors handle numbers and parenthetical expressions (highest precedence).

3.2 Lexical Analysis Implementation

The lexer performs tokenization, converting the input string into a sequence of meaningful tokens:

Listing 1: Token and Lexer Implementation

```
class Token:
    def __init__(self, type, value, position):
        self.type = type       # TOKEN_TYPES: NUMBER, PLUS, MINUS,
           MULTIPLY,
                               # DIVIDE, LPAREN, RPAREN, EOF
        self.value = value     # The actual value (number or operator)
        self.position = position

class Lexer:
    def __init__(self, text):
        self.text = text
        self.position = 0
        self.current_char = self.text[0] if text else None

    def advance(self):
        """Move to the next character in the input"""
        self.position += 1
        self.current_char = (self.text[self.position]
                             if self.position < len(self.text) else None)

    def skip_whitespace(self):
        """Skip whitespace characters"""
        while (self.current_char is not None and
               self.current_char.isspace()):
            self.advance()

    def read_number(self):
        """Read a complete number (integer or float)"""
        result = ''
        while (self.current_char is not None and
```

```
                (self.current_char.isdigit() or self.current_char == '.'
                    )):
            result += self.current_char
            self.advance()
        return float(result)

    def get_next_token(self):
        """Tokenize the next element in the input"""
        while self.current_char is not None:
            if self.current_char.isspace():
                self.skip_whitespace()
                continue

            if self.current_char.isdigit():
                return Token('NUMBER', self.read_number(), self.
                    position)

            token_map = {
                '+': 'PLUS', '-': 'MINUS', '*': 'MULTIPLY',
                '/': 'DIVIDE', '(': 'LPAREN', ')': 'RPAREN'
            }

            if self.current_char in token_map:
                char = self.current_char
                self.advance()
                return Token(token_map[char], char, self.position)

            raise SyntaxError(f"Invalid character '{self.current_char}'
                "
                              f"at position {self.position}")

        return Token('EOF', None, self.position)
```

3.3 Parser Implementation

The parser implements the recursive descent algorithm, with each grammar rule corresponding to a method:

Listing 2: Recursive Descent Parser

```
class Parser:
    def __init__(self, lexer):
        self.lexer = lexer
        self.current_token = self.lexer.get_next_token()

    def error(self, message):
        """Raise syntax error with position information"""
        raise SyntaxError(f"{message} at position {self.current_token.
            position}")

    def eat(self, token_type):
        """Consume a token of the expected type"""
        if self.current_token.type == token_type:
            self.current_token = self.lexer.get_next_token()
        else:
            self.error(f"Expected {token_type}, got {self.current_token
                .type}")

```

```
    def factor(self):
        """Parse a factor: number or parenthesized expression"""
        token = self.current_token

        if token.type == 'NUMBER':
            self.eat('NUMBER')
            return token.value

        elif token.type == 'LPAREN':
            self.eat('LPAREN')
            result = self.expression()
            self.eat('RPAREN')
            return result

        else:
            self.error("Expected number or opening parenthesis")

    def term(self):
        """Parse a term: factor followed by multiply/divide operations
           """
        result = self.factor()

        while self.current_token.type in ('MULTIPLY', 'DIVIDE'):
            op = self.current_token.type
            self.eat(op)

            if op == 'MULTIPLY':
                result *= self.factor()
            elif op == 'DIVIDE':
                divisor = self.factor()
                if divisor == 0:
                    raise ValueError("Division by zero")
                result /= divisor

        return result

    def expression(self):
        """Parse an expression: term followed by add/subtract
           operations"""
        result = self.term()

        while self.current_token.type in ('PLUS', 'MINUS'):
            op = self.current_token.type
            self.eat(op)

            if op == 'PLUS':
                result += self.term()
            elif op == 'MINUS':
                result -= self.term()

        return result

    def parse(self):
        """Parse and evaluate the complete expression"""
        result = self.expression()
        if self.current_token.type != 'EOF':
            self.error("Unexpected token after expression")
        return result
```

3.4 Calculator Integration

The calculator class integrates the lexer and parser components:

Listing 3: Calculator Class Implementation

```
class Calculator:
    def evaluate(self, expression):
        """Evaluate a mathematical expression string"""
        try:
            lexer = Lexer(expression)
            parser = Parser(lexer)
            return parser.parse()
        except (SyntaxError, ValueError) as e:
            return f"Error: {e}"

# Demonstration of calculator functionality
def demonstrate_calculator():
    calc = Calculator()
    test_cases = [
        "3 + 4 * 2",         # Expected: 11
        "(3 + 4) * 2",       # Expected: 14
        "10 / 2 + 3",        # Expected: 8.0
        "1 + 2 * 3 + 4",     # Expected: 11
        "((5 + 3) * 2) / 4" # Expected: 4.0
    ]

    for expression in test_cases:
        result = calc.evaluate(expression)
        print(f"{expression} = {result}")
```

4 Database Development Connections

4.1 Tokenization and SQL Parsing

The lexer's tokenization process directly parallels SQL parsing mechanisms. Just as we identify `NUMBER` and `PLUS` tokens, SQL parsers recognize `SELECT`, `FROM`, `WHERE`, and literal values. The position tracking we implement becomes crucial for reporting syntax errors in complex SQL statements.

The structural similarity becomes apparent when comparing our calculator's handling of `(3 + 4) * 2` with a database's parsing of `SELECT (price + tax) * quantity FROM products`. Both require identical patterns for parenthetical grouping, operator precedence, and expression evaluation.

4.2 Abstract Syntax Trees and Query Plans

Our recursive descent parser implicitly constructs an Abstract Syntax Tree (AST) through its recursive function calls. In production databases, this AST becomes the foundation for query execution plans. The precedence rules ensuring $3+4\times2$ evaluates correctly translate directly to logical rules ensuring `WHERE age > 18 AND status = 'active'` applies conditions in proper order.

4.3 Expression Evaluation Engine

The evaluation logic in our `term()` and `expression()` methods forms the foundation of database expression evaluation. When a database processes `SELECT price * 1.08 AS total_with_tax`,

it employs the same multiplication logic we implement, extended to handle column references and type conversions.

5 Advanced Error Handling

Robust error handling in calculator development establishes patterns essential for database systems:

Listing 4: Enhanced Error Handling and Recovery

```
class EnhancedCalculator(Calculator):
    def evaluate_with_context(self, expression):
        """Evaluate expression with detailed error reporting"""
        try:
            lexer = Lexer(expression)
            parser = Parser(lexer)
            result = parser.parse()
            return {
                'success': True,
                'result': result,
                'error': None,
                'suggestions': None
            }
        except SyntaxError as e:
            return {
                'success': False,
                'result': None,
                'error': f"Syntax error: {e}",
                'suggestions': self.suggest_fix(expression, e)
            }
        except ValueError as e:
            return {
                'success': False,
                'result': None,
                'error': f"Runtime error: {e}",
                'suggestions': None
            }

    def suggest_fix(self, expression, error):
        """Provide error recovery suggestions"""
        error_str = str(error)
        if "Expected RPAREN" in error_str:
            return "Check for missing closing parenthesis"
        elif "Division by zero" in error_str:
            return "Ensure divisor is not zero"
        elif "Invalid character" in error_str:
            return "Remove unsupported characters"
        return "Verify expression syntax"
```

This error handling approach scales to database query validation, where systems must catch syntax errors, type mismatches, and constraint violations while providing meaningful user feedback.

6 Database-Style Extensions

The calculator's architecture naturally extends to support database-like operations:

Listing 5: Database-Style Calculator Extension

```python
class DatabaseCalculator(Calculator):
    def __init__(self):
        super().__init__()
        self.variables = {}      # Symbol table for column references
        self.functions = {       # Built-in functions
            'ABS': abs,
            'ROUND': round,
            'MAX': max,
            'MIN': min,
            'SQRT': lambda x: x ** 0.5
        }

    def set_variable(self, name, value):
        """Simulate column values in a database row"""
        self.variables[name] = value

    def set_row_context(self, row_data):
        """Set multiple variables from row data"""
        self.variables.update(row_data)

    def evaluate_with_context(self, expression, row_data=None):
        """Evaluate expression with row context (like SQL WHERE clause)
           """
        if row_data:
            self.set_row_context(row_data)
        return self.evaluate(expression)

# Example usage demonstrating database-like functionality
def demonstrate_database_features():
    db_calc = DatabaseCalculator()

    # Simulate a database row
    row_data = {'price': 100.0, 'quantity': 3, 'tax_rate': 0.08}
    db_calc.set_row_context(row_data)

    # Expressions that would appear in SQL
    expressions = [
        "price * quantity",              # Column multiplication
        "price * quantity * (1 + tax_rate)",  # Complex calculation
        "ABS(price - 95)"                # Function call
    ]

    for expr in expressions:
        result = db_calc.evaluate(expr)
        print(f"{expr} = {result}")
```

7 Performance Analysis

Understanding the performance characteristics of our recursive descent parser provides insights directly applicable to database query optimization:

7.1 Computational Complexity

- **Tokenization**: $O(n)$ where n represents expression length

- **Parsing**: $O(n)$ for well-formed expressions
- **Evaluation**: $O(n)$ for arithmetic expressions
- **Space Complexity**: $O(d)$ where d represents maximum nesting depth

7.2 Optimization Strategies

Listing 6: Performance Optimization Techniques

```python
class OptimizedCalculator(Calculator):
    def __init__(self):
        super().__init__()
        self.expression_cache = {}    # Memoization cache
        self.compiled_expressions = {} # Pre-compiled expressions

    def evaluate_cached(self, expression):
        """Evaluate with memoization for repeated expressions"""
        if expression in self.expression_cache:
            return self.expression_cache[expression]

        result = self.evaluate(expression)
        self.expression_cache[expression] = result
        return result

    def compile_expression(self, expression):
        """Pre-compile expression for repeated evaluation"""
        lexer = Lexer(expression)
        parser = Parser(lexer)
        # In a full implementation, this would return a compiled AST
        return parser

    def clear_cache(self):
        """Clear cached results"""
        self.expression_cache.clear()
```

These optimization techniques translate directly to database query plan caching and prepared statement optimization.

8 Comprehensive Testing Framework

A robust testing framework ensures reliability and serves as a template for database system testing:

Listing 7: Comprehensive Test Suite

```python
import unittest

class TestCalculator(unittest.TestCase):
    def setUp(self):
        self.calc = Calculator()

    def test_basic_arithmetic(self):
        """Test fundamental arithmetic operations"""
        test_cases = [
            ("2 + 3", 5),
            ("10 - 4", 6),
            ("3 * 4", 12),
```

```
            ("15 / 3", 5.0)
        ]

        for expression, expected in test_cases:
            with self.subTest(expression=expression):
                result = self.calc.evaluate(expression)
                self.assertEqual(result, expected)

    def test_operator_precedence(self):
        """Test correct operator precedence handling"""
        precedence_cases = [
            ("2 + 3 * 4", 14),
            ("2 * 3 + 4", 10),
            ("10 - 6 / 2", 7.0),
            ("8 / 4 * 2", 4.0)
        ]

        for expression, expected in precedence_cases:
            with self.subTest(expression=expression):
                result = self.calc.evaluate(expression)
                self.assertEqual(result, expected)

    def test_parentheses_grouping(self):
        """Test parentheses for expression grouping"""
        grouping_cases = [
            ("(2 + 3) * 4", 20),
            ("2 * (3 + 4)", 14),
            ("((2 + 3) * 4)", 20),
            ("(10 + 5) / (2 + 3)", 3.0)
        ]

        for expression, expected in grouping_cases:
            with self.subTest(expression=expression):
                result = self.calc.evaluate(expression)
                self.assertEqual(result, expected)

    def test_error_handling(self):
        """Test proper error handling for invalid expressions"""
        error_cases = [
            "10 / 0",        # Division by zero
            "2 +",           # Incomplete expression
            "2 + + 3",       # Invalid syntax
            "2 * (",         # Unmatched parenthesis
            "2 @ 3"          # Invalid operator
        ]

        for expression in error_cases:
            with self.subTest(expression=expression):
                result = self.calc.evaluate(expression)
                self.assertTrue(str(result).startswith("Error:"))

    def test_complex_expressions(self):
        """Test evaluation of complex mathematical expressions"""
        complex_cases = [
            ("(10 + 5) * 2 - 8 / 4", 28.0),
            ("3 + 4 * 2 / (1 - 5)", 1.0),
            ("((15 / 3) + 2) * 3", 21.0)
        ]
```

```
        for expression, expected in complex_cases:
            with self.subTest(expression=expression):
                result = self.calc.evaluate(expression)
                self.assertAlmostEqual(result, expected, places=7)

# Run the test suite
if __name__ == '__main__':
    unittest.main()
```

9 Applications to Database Development

The calculator implementation establishes foundational patterns that translate directly to database system development:

1. **Query Parsing**: SQL SELECT statements employ identical parsing techniques for expression evaluation
2. **Expression Evaluation**: Column computations and WHERE clause conditions use the same evaluation engine
3. **Error Handling**: SQL syntax errors and runtime errors follow similar detection and reporting patterns
4. **Optimization**: Expression caching and AST optimization techniques apply to query plan optimization
5. **Extensibility**: The function framework supports SQL built-in function implementation

Future database development builds upon these concepts to handle table schemas, row filtering, join operations, and transaction management. The recursive descent parser becomes the SQL parser, the expression evaluator becomes the query executor, and the error handling becomes the constraint validation system.

10 Conclusion

Building a calculator with a recursive descent parser provides essential foundations for database development. The tokenization, parsing, and evaluation patterns implemented here scale directly to SQL query processing. The error handling and optimization strategies prepare developers for the complexities of database constraint validation and query optimization.

The architecture we've established demonstrates that every database query is fundamentally an expression to be parsed, validated, and evaluated—precisely what our calculator accomplishes. This foundation supports the development of production-ready database systems capable of handling complex queries with reliability and performance.

The recursive descent parser transcends academic exercise, representing a production-ready foundation that powers database query parsing in systems serving millions of applications worldwide. Through this calculator implementation, we've built not just a mathematical tool, but a cornerstone for understanding the fundamental operations that drive modern database technology.